UNIT 6: Integration Applications (Area/Volume/Cross-Sections) Highlights

AREA:

Area between f(x) and the x-axis
(where f(x) > 0)

$$A = \int_a^b f(x)\,dx$$

Area between Two Curves [f(x) and g(x)]

$$A = \int_a^b \left| f(x) - g(x) \right| dx$$

VOLUME:

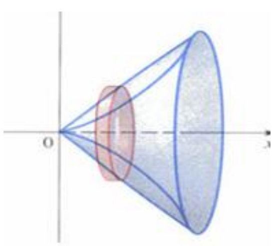
Disk Method

(Rotated about the x-axis)

$$V = \int_a^b \pi [f(x)]^2\,dx$$

(Rotated about the y-axis)

$$V = \int_a^b \pi [f(y)]^2\,dy$$



Washer Method

(Rotated about the x-axis)

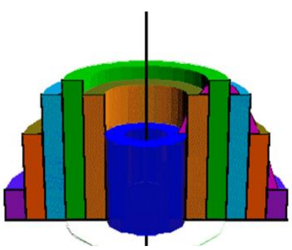
$$V = \int_a^b \pi [f(x)^2 - g(x)^2]\,dx$$

where, *f(x) is outer / g(x) is inner*

(Rotated about the y-axis)

$$V = \int_a^b \pi [g(y)^2 - f(y)^2]\,dy$$

where, g(y) is outer / f(y) is inner



Shell Method

(Rotated about the x-axis)

$$V = \int_a^b 2\pi x [f(x) - g(x)]\,dx$$

where, *f(x) > g(x)*

(Rotated about the y-axis)

$$V = \int_a^b 2\pi y [f(y) - g(y)]\,dy$$

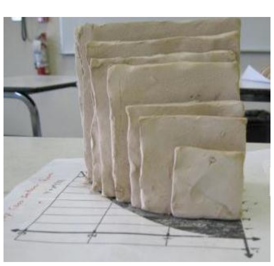
where, f(y) > g(y)

Rotations about Lines

Rotating about a Line (y = c) would resemble a rotation about the x-axis (y = 0).
Rotating about a Line (x = c) would resemble a rotation about the y-axis (x = 0).

NOTE: Adjustments to the Volume's Integral will vary according to the technique and the bounded region!

Cross-Sections (Slabs)



Commonly used Cross-Sectional Areas:

Area using squares $= s^2$

Area using equilateral triangle $= \frac{\sqrt{3}}{4} b^2$

Area using isosceles right triangle $= \frac{b^2}{2}$

Area using semi-circles $= \frac{\pi r^2}{2}$

($\perp$ to the x-axis)

$$V = \int_{x=a}^{x=b} A(x)\,dx$$ where A(x) is the $\text{area}_{\text{cross section}}$

($\perp$ to the y-axis)

$$V = \int_{y=a}^{y=b} A(y)\,dy$$ where A(y) is the $\text{area}_{\text{cross section}}$